Quadratic Optimal Control Problems:
Discretizations and Error Estimates

Subject of the talk is a class of quadratic optimal control problems governed by elliptic differential equations. For a discretization of the control problem we first discretize the differental equation by a finite element method. Then we approximate the control functions u by piecewise polynomials $P_h(u)$ from a finite dimensional subspace to obtain a full discretization of the problem. Here, h denots the meshsize of the grid used for the discretizitaion. For the solution of the discretized problem we first derive discrete error estimates of order 2, i.e., estimates of the type $\|u_h^* - P_h(u^*)\|_h \leq c\,h^2$, where u_h^* is the solution of the discretized problem, u^* is the solution of the continuous problem and $\|\cdot\|_h$ is the norm of the finite dimensional subspace. Unfortunately, the global error $\|u_h^* - u^*)\|_\infty$ is only of order 1. We therefore construct a new control, for which we can show global convergence of order 2. In the last part of the talk we discuss extensions of the results to finite difference discretizations of elliptic control problems and to Runge-Kutta discretizations of general class of quadratic control problems.

Parts of the talk are joint work with my PhD student Nils Bräutigam and Arnd Rösch from RICAM, Linz.