

Capital Growth: Theory and Practice*

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Abstract

In capital accumulation under uncertainty, a decision-maker must determine how much capital to invest in riskless and risky investment opportunities over time. The investment strategy yields a stream of capital, with investment decisions made so that the dynamic distribution of wealth has desirable properties. The distribution of accumulated capital to a fixed point in time and the distribution of the first passage time to a fixed level of accumulated capital are variables controlled by the investment decisions. An investment strategy which has many attractive and some not attractive properties is the growth optimal strategy, where the expected logarithm of wealth is maximized. This strategy is also referred to as the Kelly strategy. With the Kelly strategy, the first passage time to arbitrary large wealth targets is minimized, and the probability of reaching those targets is maximized. Moreover, the time to reach sufficiently large wealth goals is minimized. However, the strategy is very aggressive since the Arrow-Pratt risk aversion index is essentially zero. Hence, the chances of losing a substantial portion of wealth are very high, particularly if the estimates of the returns distribution are in error. In the time domain, the

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chances are high that the first passage to subsistence wealth occurs before achieving the established wealth goals.

This chapter is a survey of the theoretical results and practical uses of the capital growth approach. It is a companion to the chapter in this volume on the Kelly criterion by E. O. Thorp. Alternative formulations for capital growth models in discrete and continuous time are presented. Various criteria for performance and requirements for feasibility are related in an expected utility framework. Typically, there is a trade-off between growth and security with a fraction invested in an optimal growth portfolio determined by the risk aversion criteria. Models for calculating the optimal fractional Kelly investment with alternative performance criteria are formulated. The effect of estimation and modeling error on strategies and performance is discussed.

Various applications of the capital growth approach are made to futures trading, lotto games, horseracing, and the fundamental problem of asset allocation between stocks, bonds and cash. The chapter concludes with a discussion of some of the great investors and speculators, and how they used Kelly and fractional Kelly strategies in their investment programs.

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