A Very Basic Mixed Integer Set: Mixing and its Applications

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Very little is known about the polyhedral structure of even simple mixed integer programs. Here we study a very simple set, called the mixing set. This set and valid inequalities for it were proposed by Gunluk and Pochet in an effort to explain and understand why the same procedure of "mixing mixed integer rounding (MIR) inequalities" gave valid inequalities for many different variants of the constant capacity lot-sizing problem, and subsequently Miller and Wolsey took this abstraction a little further by considering the intersection of mixing sets plus certain additional constraints.

In this talk we pursue this effort at abstraction by considering two "abstract sets"

$$x_i + z_i \geq b_i \text{ for } i \in M_1$$
$$x_i + z_i \leq c_i \text{ for } i \in M_2$$
$$x \in X(\alpha), z \in X(\beta)$$
$$x \in \mathbb{R}^m, z \in \mathbb{Z}^m$$

where

$$X(\gamma) = \{y \in \mathbb{R}^m : y_i - y_j \leq \gamma_{ij} \ \forall\ i, j, l_i \leq y_i \leq u_i \ \forall\ i\},$$

and a flow variant of the mixing set

$$x_i + w_i \geq b_i \text{ for } i \in M$$
$$0 \leq w_i \leq z_i \text{ for } i \in M$$
$$z \in X(\beta)$$
$$x, w \in \mathbb{R}^m, z \in \mathbb{Z}^m$$

We show that, modulo a slight restriction, the convex hulls of both sets are obtained with mixing inequalities. This is joint work with Michele Conforti and Marco Saturni.

Turning back to lot-sizing, we show that a constant capacity problem with production time windows, and surprisingly both a multi-item problem with a joint set-up variable, and a class of single item problems with varying capacities can also be solved by mixing. This latter is joint work with Yves Pochet.