Copositive programming and combinatorial optimization

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We consider linear optimization with completely positive matrices, given as follows,

$$(CP) \quad \min\langle C, X\rangle \text{ such that } A(X) = b,\ X = VV^T,\ V \geq 0.$$

Many NP-hard combinatorial optimization problems have a formulation of this type, hence it is difficult to solve this problem.

In the first part, I will explain such formulations for the stable set problem and for graph coloring. It turns out that in fact any 0-1 optimization problem (with quadratic objective function and linear constraints) can equivalently be formulated as (CP), a recent result of Sam Burer (2007).

In this situation, it is interesting to either approximate the cone of completely positive matrices 'from outside' in order to get relaxations of (CP). But it is also a challenge to approximate this cone from the inside, in order to get near optimal (feasible) solutions (primal heuristic) for CP.

In the second part of the talk I will touch on both issues. Outer approximations were recently suggested by Parrilo, by providing 'inner approximations' of the dual cone of copositive matrices. I will briefly explain the main ideas of this approach, and the practical difficulties.

Finally, I will address the issue of heuristic solutions to (CP). We investigate heuristics to solve this problem, which work in the V-space of nonnegative matrices. The resulting quadratic objective function is regularized and the constraints are linearized. To reach feasibility, it is necessary to use some form of corrector iterations. The main effort will be reduced to solving a convex quadratic problem in sign constrained variables.

To test our algorithm we also propose a generator for instances of (CP), where the optimal solution value is known, and which are nontrivial in the sense that the optimal solution X is not attained at a positive definite and elementwise nonnegative matrix.

We present some preliminary computational results on randomly generated instances as well as on some copositive formulations of combinatorial optimization problems.

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