ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Institute for Operations Research
ETH Zurich HG G21-23
Jan Foniok jan.foniok@ifor.math.ethz.ch
David Adjiashvili david.adjiashvili@ifor.math.ethz.ch

IFOR

Nonlinear Discrete Optimization — Exercise 5

Exercise 1: Multi-Commodity Transshipment

In this exercise we would like to solve a Multi-Commodity Transshipment problem (Section 4.4.1 in the lecture notes) with 3 commodities $C = \{1, 2, 3\}$. The graph we will consider is the following direction of K_4.

- $V = \{1, 2, 3, 4\}$
- $E = \{(1, 2), (1, 4), (2, 3), (2, 4), (3, 1), (4, 3)\}$

where the edge (i, j) is directed from i to j. In addition, vertex i is a supplier of 3 units of commodity i, for $i \in \{1, 2, 3\}$ (vertex 4 has no supply). Furthermore vertex i has a demand of 1 unit of every commodity *except* for commodity i. The capacities on all edges are 9. Let $f : E \times C \to \mathbb{Z}_+$ be a flow. Then the value of the flow is defined in the following way

$$g(f) = \sum_{e\in E} (\sum_{c\in C} f(e, c))^2.$$

We are interested in a legal flow that minimizes g.

(a) Find a feasible solution which does not use the edge $(2, 3)$ (i.e., the flow on this edge is 0 for all commodities).

(b) Use the theory of n-fold integer programming to solve the minimization problem (Use the construction in Section 4.4.1).

Exercise 2: Multiway Tables

(a) We consider all 3-dimensional nonnegative integer tables $\mathbb{Z}_+^{3\times3\times3}$ with all side lengths equal to 3. We fix all possible 27 line sums to be 9. Notice that trivially the table

$$x_{i,j,k} = 3$$

for all $i, j, k \in \{1, 2, 3\}$ satisfies these line sums. Find the spectrum of values for the entry $x_{1,1,1}$ for these line sums.

(b) Solve the same problem for tables in $\mathbb{Z}_+^{n\times n\times n}$ with line sums all equal to n (advanced).

(c) In this exercise we would like to use the Universality construction to generate a table with a given spectrum. Let $I = \{10, 11, ..., 100\}$.

- Derive an integer program over the variables $y_0, .., y_r$ for a sufficiently large r, for which the set of feasible solutions restricted to the variable y_0 is exactly I (Hint: $r = 2$ is sufficient).
- Use Step 2 of the universality construction to generate m, n, t and plane sums for a $m \times n \times t$ table which corresponds to the polytope defined in the previous part.

Due date: Please hand in the solution before **Tuesday, 02.06.2009, 13:00** in the Discrete Optimization tray at HG G21.
Certificate condition: At least 50% of the exercises have to be solved.