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IFOR

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Integer Programming — Assignment 9

http://www.ifor.math.ethz.ch/teaching/lectures/integer_prog_ss10

Exercise 1: Valid Inequalities of a Polyhedron

Let $P = \{x \in \mathbb{R}^n : Ax \leq b, x \geq \mathbf{0}\}$ where $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$ be an empty polyhedron. Let $d^T x \leq d_0$ be any inequality. Show that if $\{u \in \mathbb{R}^m : u^T A \geq d^T, u \geq \mathbf{0}\}$ is not empty, there exists $u \in \mathbb{R}^m, u \geq \mathbf{0}$ such that $d^T x \leq d_0$ is equivalent to or dominated by $(u^T A)x \leq u^T b$.

Exercise 2: Disjunctive Constraints

Let $S \subseteq \mathbb{Z}^n$. Show that if r, s are positive numbers such that both $d^T x + r(x_i - l) \leq d_0$ and $d^T x - s(x_i - l + 1) \leq d_0$ are valid for S, then $d^T x \leq d_0$ is valid for S.

Exercise 3: Word Building Game continued

Recall the word building game from assignment three. You are given the words w_1 :="integer", w_2 :="progamming", w_3 :="equation" and w_4 :="system". The inventory of tiles is restricted as follows:

letter α	number of tiles N_α
"e"	3
"n"	2
"g"	2

For every other letter there is no restriction on the number of tiles. Constructing the word w_1 gives $p_1 := 6$ points, for the others, we have $p_2 := 6$, $p_3 := 3$ and $p_4 := 3$. If you construct both w_1 and w_2, you get $b_{12} := 2$ bonus points. You also get $b_{34} := 2$ additional points for constructing w_3 and w_4. There is no bonus for any other combination. Respecting the letter availability constraints, construct words so that the number of points is maximized. Formulate the corresponding IP, then solve the LP-relaxation using LP_SOLVE. Continuously add disjunctive constraints until the optimal solution becomes integral.

Please hand in your assignment no later than **Tuesday, 18.05.2010, 15:00** at HG G 21 ("Integer Programming" tray).
Certificate condition: At least 50% of the exercises have to be solved.