Eidgenössische Technische Hochschule Zürich
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Integer Programming — Assignment 12

http://www.ifor.math.ethz.ch/teaching/lectures/integer_prog_ss10

Exercise 1: Lagrangian Duality
Consider the problem

$$
\text{(IP)} \qquad \begin{array}{lrl}
\max & 2x_1 + 5x_2 & \\
\text{s.t.} & 4x_1 + x_2 & \leq 28, \\
& x_1 + 4x_2 & \leq 27, \\
& x_1 - x_2 & \leq 1, \\
& x & \in \mathbb{Z}^2_+.
\end{array}
$$

Graphically demonstrate the following properties:

(i) if any two constraints are dualized, the value of the Lagrangian dual equals the optimal value of the LP relaxation.

(ii) there is an objective function such that (i) is wrong.

(iii) if any single constraint is dualized, the value of the Lagrangian dual is an improvement on the optimal value of the LP relaxation.

Exercise 2: Generalized Assignment Problem
Consider three different Lagrangian duals for the generalized assignment problem:

$$
\text{(IP)} \qquad \begin{array}{lrll}
\max & \sum_{j=1}^{m} \sum_{i=1}^{n} p_{ij} x_{ij} & & \\
\text{s.t.} & \sum_{i=1}^{n} x_{ij} & \leq 1, & j = 1, \dots, m \\
& \sum_{j=1}^{m} l_{ij} x_{ij} & \leq b_i, & i = 1, \dots, n \\
& x & \in \{0,1\}^{m \times n}. &
\end{array}
$$

Discuss their relative merits according to the following three criteria:

(i) ease of solution of the Lagrangian subproblems,

(ii) ease of solution of the Lagrangian dual, and

(iii) the strength of the resulting Lagrangian dual bound.

Please hand in your assignment no later than **Tuesday, 01.06.2010, 15:00** at HG G 21 ("Integer Programming" tray).
Certificate condition: At least 50% of the exercises have to be solved.