

Spring Semester 2010

Integer Programming — Assignment 1

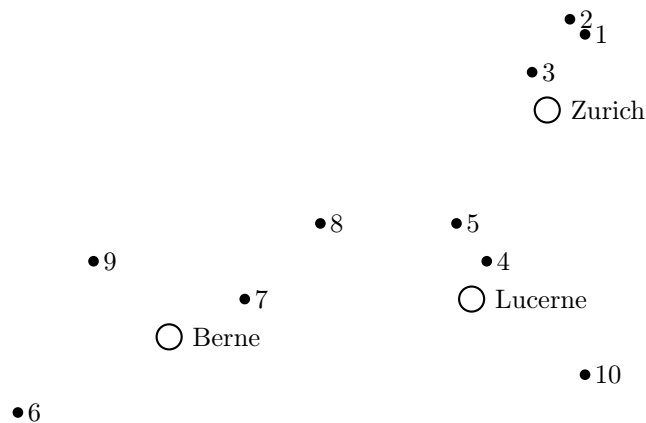
http://www.ifor.math.ethz.ch/teaching/lectures/integer_prog_ss10

Exercise 1: Facility Location Problem

In modeling distribution systems, decisions must be made about trade-offs between transportation costs and costs for operating warehouses. Let a set of n warehouse locations and a set of m clients be given. Assume that each warehouse i , if opened, has fixed operating costs o_i . Further, let c_{ij} be the per-unit costs to ship the client's j demand d_j from warehouse i . The decisions to be made are which warehouses to operate and how much to ship from each warehouse to each customer.

a) Model this problem as an IP.

b) Suppose some company's plant with a nearby warehouse is located in Zurich. There are 10 regular clients (black points in the below picture) with demands $d = (1 \ 3 \ 2 \ 4 \ 1 \ 7 \ 2 \ 3 \ 5 \ 1)$ of some good. So far, every client is served from Zurich. The company's management division thinks about using remote warehouses for the purpose of serving some clients from those locations. Berne and Lucerne are under consideration. Estimations for operating costs (in monetary units) of those locations are $o_b = 15$ for Berne, $o_l = 7$ for Lucerne respectively. For the per-unit transportation costs, we have $c_z = (1.5 \ 1.5 \ 0.5 \ 2 \ 2 \ 8 \ 5 \ 3.5 \ 6.5 \ 4)$ to ship from Zurich, $c_b = (8 \ 8 \ 7 \ 5.5 \ 5 \ 3 \ 2 \ 3 \ 2.5 \ 6.5)$ for Berne and $c_l = (5 \ 5 \ 4.5 \ 1 \ 1.5 \ 7.5 \ 4 \ 3.5 \ 6 \ 2.5)$ for Lucerne. Use LP_SOLVE to decide whether it makes sense to operate additional warehouses in Berne and/or Lucerne. Sketch the optimal solution in the picture.



Exercise 2: Assignment Problem

In the lecture you have seen that the Assignment Problem can be formulated as a binary IP. The feasible region P of its LP relaxation is of the form

$$P := \{x \in \mathbb{R}^n : Ax = b, x \geq 0\},$$

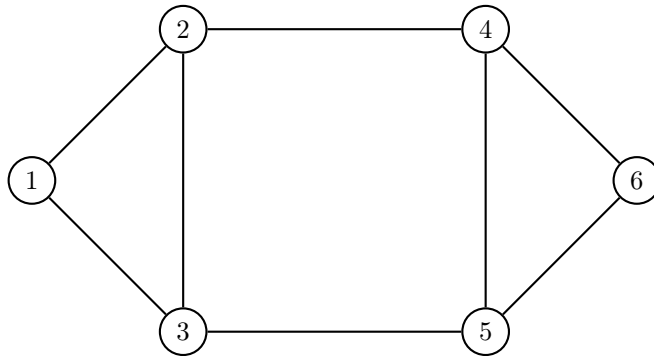
where A is a 0/1 matrix of a special kind and b is the all-one vector.

a) Show that A is a totally unimodular matrix, that is, every square submatrix has determinant equal to 0, 1 or -1 .

b) Using a), prove that the feasible region P is integral.

Exercise 3: Optimal Perfect Matching Problem

a) Consider the following weighted undirected graph $G = (V, E)$. The actual weights play no role, so they are not depicted.



Formulate the Optimal Perfect Matching Problem for G as a binary IP and let P be the feasible region of the corresponding LP relaxation. Give all extreme points of P .

b) Prove that the feasible region arising from a general graph is half-integral, that is, the components of every extreme point are in $\{0, \frac{1}{2}, 1\}$.

Please hand in your assignment no later than **Tuesday, 02.03.2010, 15:00** at HG G 21 ("Integer Programming" tray).
Certificate condition: At least 50% of the exercises have to be solved.